Math Olympics
Problem Set
2021

Thank you for your interest in the Middle Georgia State University Math Olympics. As you know, we suspended the competition this year due to COVID-19. However, we didn't think that should stop you from enjoying some cool math and maybe exploring some new ideas! We hope you enjoy these problems, and we hope to see you at next year's competition!

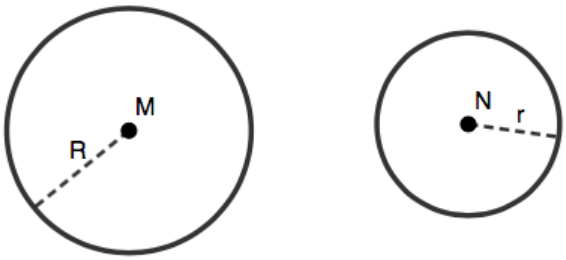
1. For natural numbers k and m, if n = km + 1 objects are distributed among m sets, then the pigeonhole principle asserts that at least one of the sets will contain at least k + 1 objects. Suppose you have a bag containing 33 red marbles, 33 white marbles, and 33 blue marbles. What is the minimum number of marbles you would have to choose randomly from the bag to ensure that you get 5 marbles of the same color?

2. There is a classroom that is 12 ft by 12 ft. What is the minimum number of people we can fit into the classroom so that we know there are at least two people within $4\sqrt{2}$ ft (approximately 5.66 ft) of each other?

3. The variance of a dataset is the average squared distance from the mean. Given 4 fixed values x_1 to x_4, what value can be given to x_5 in order to minimize the variance of $\{x_1, \cdots x_5\}$.

4. There are n candidates and m voters in an election. A news station reports the final count with a table showing how many votes each candidate received. How many different tables are possible?

5. Given two circles centered at M and N with radius R and r below:

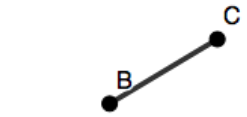


use compass and straight edge only to find a point P such that:

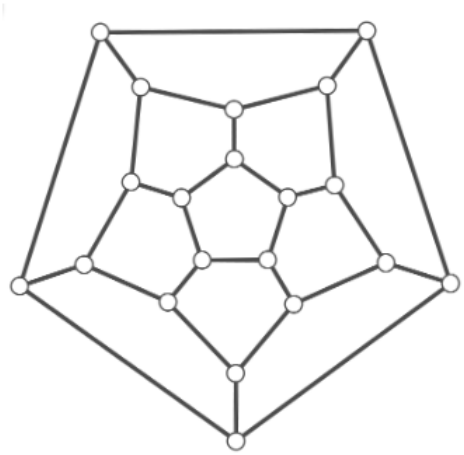
$$(PM - R)(PM + R) = (PN - r)(PN + r)$$

You might want to check out the Intersecting Secants Theorem !

6. Given a point A and a segment BC, use a compass and straight edge to construct a segment starting at A with length BC.

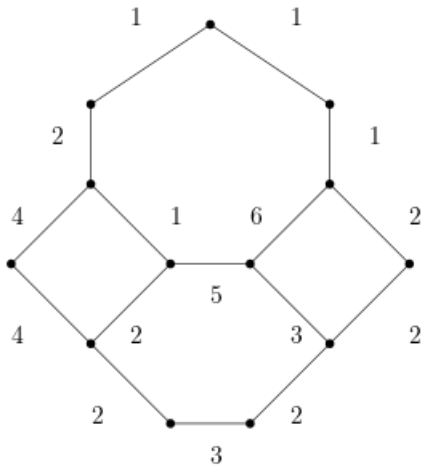


7. In the field of Graph Theory, a node is called a vertex and an edge is a line that connects any 2 vertices. A walk is a finite or infinite sequence of edges which joins a sequence of vertices. A trail is a walk in which all edges are distinct (no repeats). A path is a trail in which all vertices are distinct. Consider the graph below:



Is it possible to create a path, using the following diagram by including every vertex? (remember you can't reuse vertices or edges)

8. A spanning tree of a graph G is a subgraph of the graph with as many edges removed as possible, but leaving the graph still connected (i.e. there is path between any two vertices). Edges in graphs can also be assigned numbers, known as weights of costs. As a result, varying spanning trees of a graph can have different costs. Given the following graph:



what is the cost of a minimum cost spanning tree? You might want to check out Kruskal's Algorithm. (Note that the cost is unique, but there may be different minimum cost spanning trees)

9. Say you find you and your best friend playing blackjack in Vegas. With you, your best friend and the dealer being the only 3 people at the table, and this dealer is dealing with a double deck (so a combined deck of two Standard 52 Card decks of playing cards). These are the cards you can see:

If your friend went first and chose to stay, what is the probability that the dealer has a 10, J, Q, or K, so you end up winning?

10. The famous Monty Hall problem: Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say Door Number 1, and the host, who knows what's behind each of the doors, opens another door, say Door Number 3, which reveals a goat. He then says to you, "Do you want to switch to another door?" Is it to your advantage to switch your choice?

    (a) Now, suppose you have a standard deck of playing cards with one single Joker card added. As the host shuffles the deck of 53 cards fairly, he asks you to pick a card, if you end up picking the single Joker card, you win $1 million. After you pick a card, the host flips over 51 other cards, which are all not the Joker. Do you want to stay with your initial card choice? Or switch to the last remaining card?

    (b) What is your probability of winning the million dollars by staying? What is the expected value for choosing to stay?